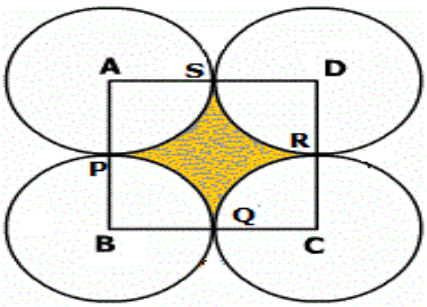
12. Areas Related to Circle

Question-1

Four equal circles are described about the four corners of a square so that each touches two of the others. The shaded area enclosed between the circles being 24/7 sq. cm, find the radius of the circles. [Use = $\frac{22}{7}$].

Solution:

Let A, B, C and D be the four corners of the given square. Let the radius of each circle be 'a' cm. so
that the side of the square is 2a cm. Let the circles touch in pairs at P, Q, R and S respectively.



Now area of the sector APS = $(\frac{90}{360})$ a^2 sq. cm. = $(\frac{1}{4})$ π a^2 sq.cm.

Similarly area of each sector APS, BPQ, CQR and DRS = (1/4) π a^2 sq.cm.

Now area enclosed by the circles = Area of the square - sum of the areas of the sectors APS, BPQ, CQR and DRS.

= $[(2a)^2 - 4 \times (1/4)\ \pi\ a^2]$ sq.cm.

= $(4 -)a^2$ sq. cm.

= $0.857a^2$ ----------(i)

But area enclosed by the circles = 24/7 sq.cm. (given) ---------(ii)

From (i) and (ii),

$0.857\ a^2 = 24/7$

$a^2 = \frac{24}{7 \times 0.857} = 4$

a = 2 cm.

Hence the radius of the circle is 2 cm.

Question-2

A boy is cycling such that the wheels of the cycle are making 140 revolutions per minute. If the diameter of the wheel is 60 cm, calculate the speed per hour with which the boy is cycling.

Solution:

The diameter of the cycle wheel = 60 cm.

Hence radius is 60/2 = 30 cm.

The circumference of the wheel = 2π r = $2 \times (\frac{22}{7}) \times 30$ cm = $\frac{2 \times 22 \times 30}{7 \times 100 \times 1000}$ km

Hence the distance covered in 1 revolution = the circumference of the

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wheel = $\frac{2\times 22\times 30}{7\times 100\times 1000}$ km.

∴ The distance covered in 140 revolutions = $\frac{2\times 22\times 30}{7\times 100\times 1000}\times 140$

i.e in 1min.(1/60hr.) the distance covered = $\frac{2\times 22\times 30}{7\times 100\times 1000}\times 140$

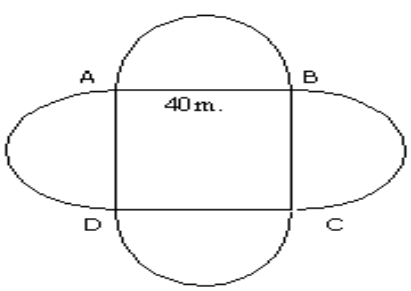
The distance covered in 1hr. = $\frac{2\times 22\times 30}{7\times 100\times 1000}\times 140\times 60 = 15.84$ km.

∴ The cycling speed of the boy = 15.84 km/hr.

Question-3

The square water tank has its sides equal to 40 m. There are four semi-circular grassy plots all round it. Find the cost of surfing the plots at Rs.1.25 per sq.m. (Use p = 3.14).

Solution:



ABCD is the square plot of side = 40m.

The area of the four semi circular plots = area of two circles with radius 20 m.(40/2 m).

Hence area of the four semi circular plots = $2\times \pi (20)^2$.

$= 800\ \pi$ sq.m.

The cost of turfing the plot at Rs. 1.25 / sq. m. = $800\pi \times 1.25$

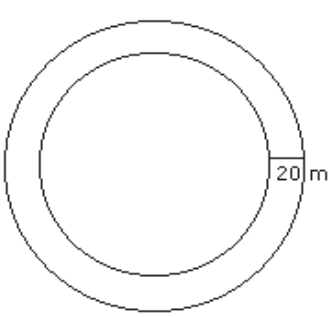
$= 800 \times 3.14 \times 1.25$

= Rs. 3140.

Question-4

The circumference of a circular park is 314 m. A 20 m. wide concrete track runs round it. Calculate the cost of laying turf in the park at `1.25 per sq.m. and the cost of concrete track at `1.25 per sq. m.

Solution:



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The circumference of the circular plot = 314 m.

$2\pi r = 314$

$r = \frac{314}{2 \times 3.14} = 50$ m.

The area of the plot = $\pi r^2 = 3.14 \times (50)^2$

$= 3.14 \times 2500$

$= 7850$ sq. m.

The cost of laying turf at `1.25 /sq. m. = 7850×1.25

= `9812.50

Radius of the bigger circle R = radius of the park + width of the concrete track

= 50 + 20 = 70 m.

Hence area of the concrete track = $\pi (R^2 - r^2)$

$= \pi (70^2 - 50^2)$

$= 3.14(70 + 50)(70 - 50)$

$= 3.14 \times 120 \times 20$

$= 7536$ sq.cm.

Cost of laying the concrete track at `1.25 = $7536 \times 1.25$ = `9420.

Question-5

The diameter of a cycle wheel is 70 cm. Find how many times the wheel will revolve in order to cover a distance of 110 m.

Solution:

The diameter of the cycle wheel = 70 cm

∴ The circumference of the wheel = πd

$= \frac{22}{7} \times 70$

$= 220$ cm

The wheel will cover a distance of 220 cm by revolving once.

The distance to be covered by the wheel = 110 m = 11000 cm

∴ Number of times the wheel will revolve to cover 110 m = $\frac{11000}{220}$

$= 50$

∴ Number of times the wheel will revolve to cover 110 m is 50.

Question-6

An ox in a kolhu (an oil pressing apparatus) is tethered to a rope 3 m long. How much distance does it cover in 14 rounds?

Solution:

The distance covered by the ox is the circumference of the circle with radius equal to the length of the rope.

Radius of the rope = 3 m

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∴ The circumference of the circle = $2\pi r$

$= 2 \times \frac{22}{7} \times 3$ m

The distance it covers in 14 rounds = $2 \times \frac{22}{7} \times 3 \times 14$

= 264 m.

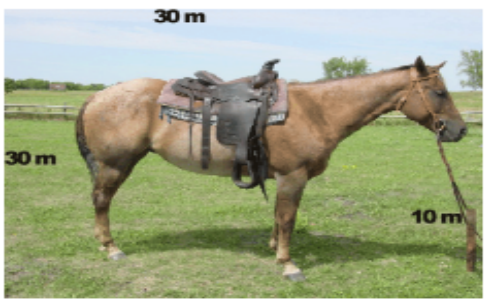
The distance it covers in 14 rounds is 264 m.

Question-7

A horse is tied to a pole fixed at one corner of a 30 m × 30 m square field of grass, by means of a 10 m long rope (figure).
[Take π = 3.14.]

(i) Find the area of that part of the field in which the horse can graze.

(ii) Find the increase in the grazing area if the rope were 20 m long instead of being 10 m long.



Solution:

(i) The area of the field in which the horse can graze is one-fourth of a circle with radius 10 m.

Radius of the portion of the circle = 10 m

∴ Area of the field in which the horse can graze = $\frac{1}{4} \times \pi r^2$

$= \frac{1}{4} \times 3.14 \times 10 \times 10$

$= \frac{314}{4}$

$= 78.5\ m^2$

∴ Area of the field in which the horse can graze is 78.5 m^2.

(ii) The radius of the circular portion = 20 m,

∴ Area of the field in which the horse can graze = $\frac{1}{4} \times \pi r^2$

$= \frac{1}{4} \times 3.14 \times 20 \times 20$

$= \frac{314 \times 4}{4}$

$= 314\ m^2$

∴ Increase in the area of the field in which the horse can graze = (314 - 78.5) m^2

$= 235.5\ m^2$

∴ The increase in the grazing area if the rope were 20 m long instead of being 10 m long is 235.5 m^2.

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Question-8

The radius of a circle is doubled. What is the ratio of the areas of the new circle to the area of the given circle?

Solution:

Let the radius of the circle be r units.

Area of the circle = πr^2

The radius of the circle when it is doubled = 2r

Area of the circle when the radius is doubled = $\pi (2r)^2$

$= 4\pi r^2$

$\therefore$ The ratio of the areas of the new circle to the given circle = $4\pi r^2 : \pi r^2$

$= 4 : 1.$

$\therefore$ The ratio of the areas of the new circle to the given circle is 4 : 1.

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